ATM2BT
Unified approach to turbulence

Strongly Non-linear Fluid Dynamics of Shear-Flow Convection

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Outreach talk

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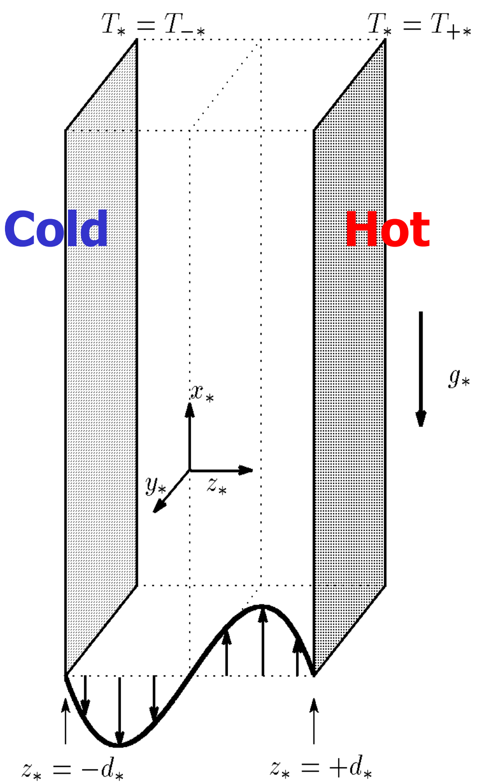
Background



Basic conductive state

experiments
Vest & Arpaci(1969): air, oil
Hart(1971): air, water

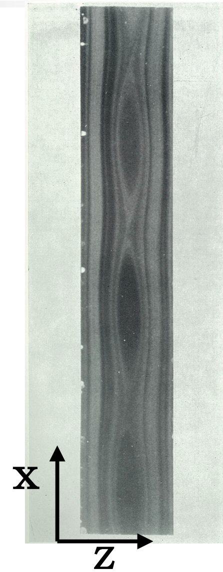
Secondary flow (2D transverse rolls)

Monotone instability

Tertiary flow

Oscillatory instability

PRESENT STUDY



theory
Nagata & Busse(1983): Pr=0
Chait & Korpela(1989): Pr=0.71, 1000
Clever & Busse(1995): Pr=0.71, 7.0

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Agenda

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- 1. Mathematical formulation
- 2. Usage of Navier-Stokes equation for the Linear stability analysis for various fluids (different Pr values)
- 3. The five cases to be studied
- 4. The software to be used
- 5. Summary of your results

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Agenda

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- 1. Present work is applied to Double Glazing, Atmospheric Physics, Natural Convection, earth interior physics
- 2. The 5 different cases to be studied are the following ones:

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The five cases

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- Inner Wall Hot, Outer Wall Cold Pr=0.7
- Inner Wall Cold, Outer Wall Hot Pr=0.7
- Inner Wall Hot, Outer Wall Cold Pr=26
- Inner Wall Hot, Outer Wall Cold Pr=1000
- Application of streamwise pressure gradient (Re=finite, see below), Pr=1000.

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Agenda

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- 1. Usage of Fortran
- 2. Executable for the 5 cases provided
- 3. Find Stability (neutral) curve, as in following example
- 4. Summarise and present your results

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Mathematical theory

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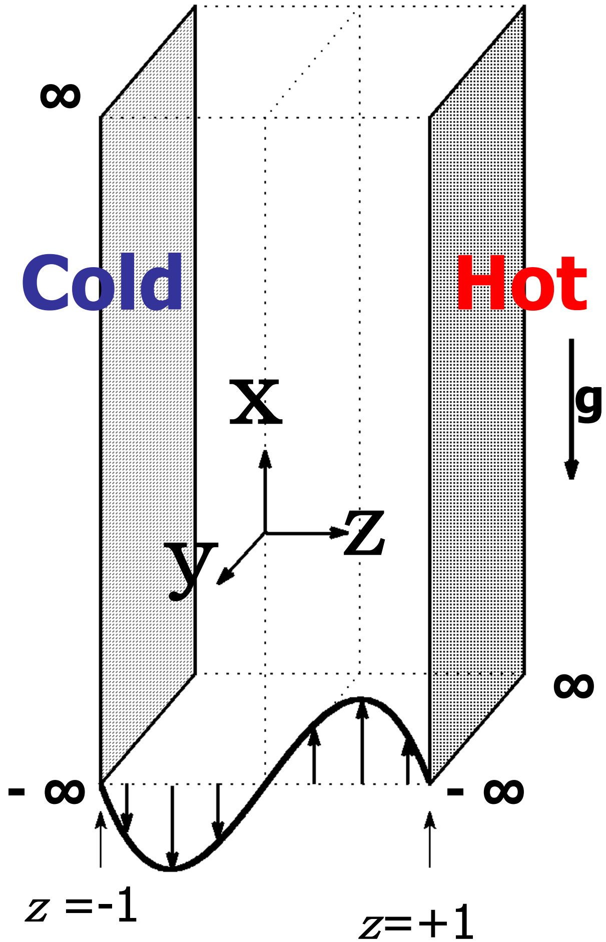
Viscous incompressible fluids with Boussinesq approximation

Equation of continuity: $$\nabla \cdot \boldsymbol{u} = 0$$

Equation of momentum: $$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla \Pi + \theta \boldsymbol{i} + \nabla^2 \boldsymbol{u}$$

Equation of energy: $$\frac{\partial \theta}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta = Pr^{-1} \nabla^2 \theta$$



Boundary Condition: No-slip, Fixed temperatures

Velocity =0 at the boundaries

$$\theta = \pm Gr$$

Equation of state: $$\rho_* = \rho_0 \{1 - \gamma (T_* - T_{0*})\}$$

γ : coeff. of thermal expansion

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Mathematical theory

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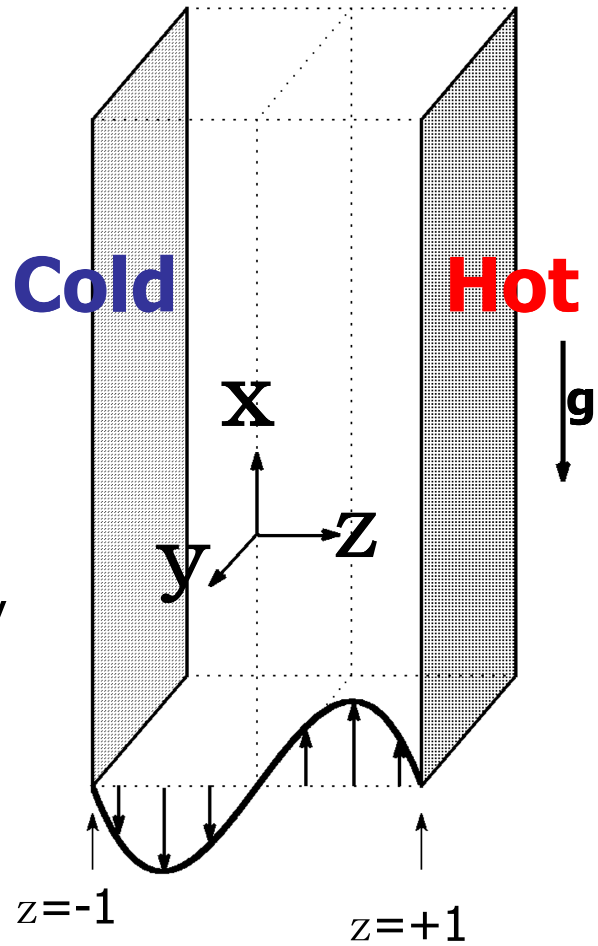
Non-dimensional parameters

Prandtl number: $Pr = \frac{\nu_*}{\kappa_*}$

Grashof number: $Gr = \frac{\gamma_* g_* d_*^3 (T_{+*} - T_{-*})}{2{\nu_*}^2}$

κ :thermal diffusivity

ν :kinematic viscosity



Basic state

$$\boldsymbol{U_B}(z) = \frac{Gr}{6} z(1 - z^2)\boldsymbol{i} + \mathrm{Re}\,\boldsymbol{i}$$

$$T_B(z) = Grz$$

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Linear analysis

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perturbations

$$\boldsymbol{u} = U_B(z)\boldsymbol{i} + \check{\boldsymbol{u}} \qquad \theta = T_B(z) + \check{\theta} \qquad \Pi = \Pi_B + \check{\Pi}$$

toroidal and poloidal decomposition

$$\check{\boldsymbol{u}} = \nabla \times \nabla \times (\phi \boldsymbol{k}) + \nabla \times (\psi \boldsymbol{k})$$

→ Eq. of continuity is satisfied

Operation of k·(∇× and k·(∇×∇× on Eq. of momentum

$$\partial_t \triangle_2 \psi + \{U(z)\partial_x - \nabla^2\}\triangle_2\psi - U'\partial_y \triangle_2 \phi - \partial_y \check{\theta} = 0$$

$$\partial_t \nabla^2 \triangle_2 \phi + \{U(z)\partial_x - \nabla^2\}\nabla^2\triangle_2\phi - U''\partial_x \triangle_2 \phi - \partial^2_{xz} \check{\theta} = 0$$

$$\partial_t \check{\theta} + U(z)\partial_x \check{\theta} - Gr\triangle_2\phi - Pr^{-1}\nabla^2\check{\theta} = 0$$

Boundary Conditions:

$$\phi = \frac{\partial \phi}{\partial z} = 0$$

$$\psi = \check{\theta} = 0$$

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Linear analysis

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Normal mode

$$\phi = \Phi(z)\exp\{i(\underline{\alpha} x + \underline{\beta} y) + \underline{\sigma} t\}$$

$$\psi = \Psi(z)\exp\{i(\underline{\alpha} x + \underline{\beta} y) + \underline{\sigma} t\}$$

$$\breve{\theta} = \Theta(z)\exp\{i(\underline{\alpha} x + \underline{\beta} y) + \underline{\sigma} t\}$$

$$\Phi(z) = \sum_{l=0}^{L} a_l (1-z^2)^2 T_l(z)$$

$$\Psi(z) = \sum_{l=0}^{L} b_l (1-z^2) T_l(z)$$

$$\Theta(z) = \sum_{l=0}^{L} c_l (1-z^2) T_l(z)$$

α : wavenumber in x-direction

β : wavenumber in y-direction

σ : growth rate

Eigenvalue problem: $$A\boldsymbol{x} = \sigma B\boldsymbol{x}$$

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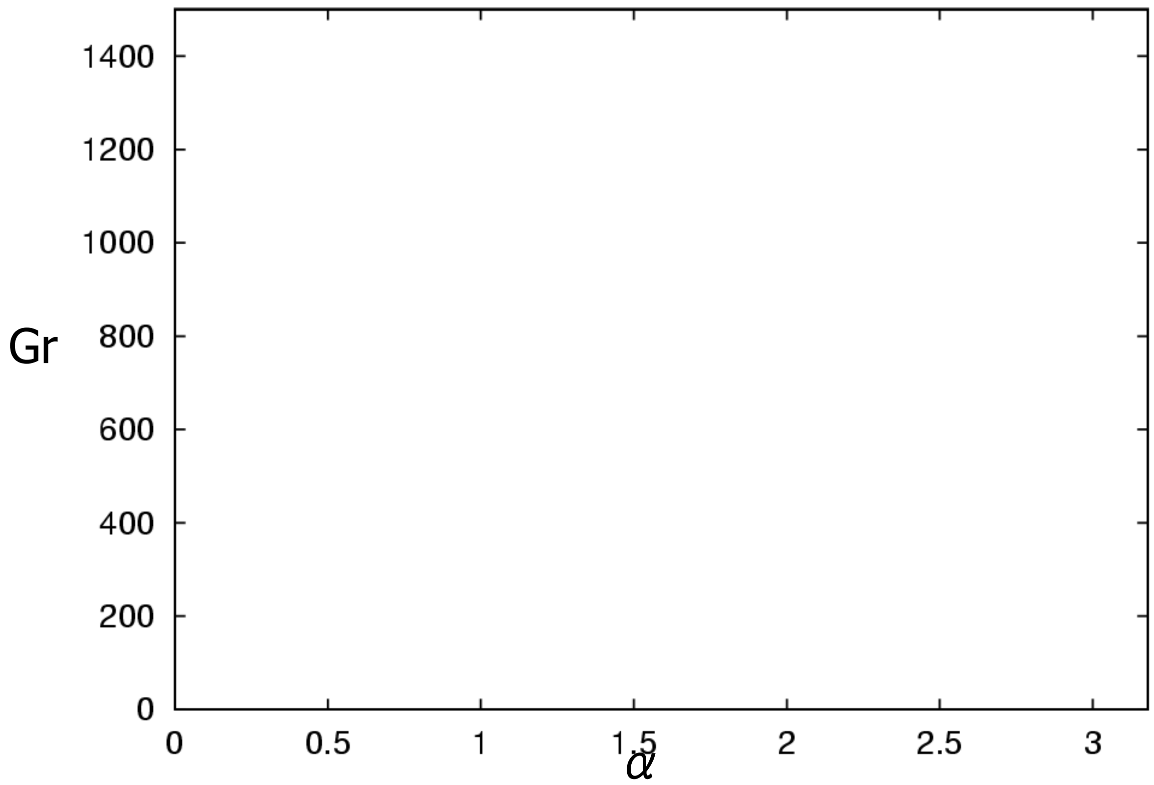
Results

Pr=0.00001 (liquid metals)



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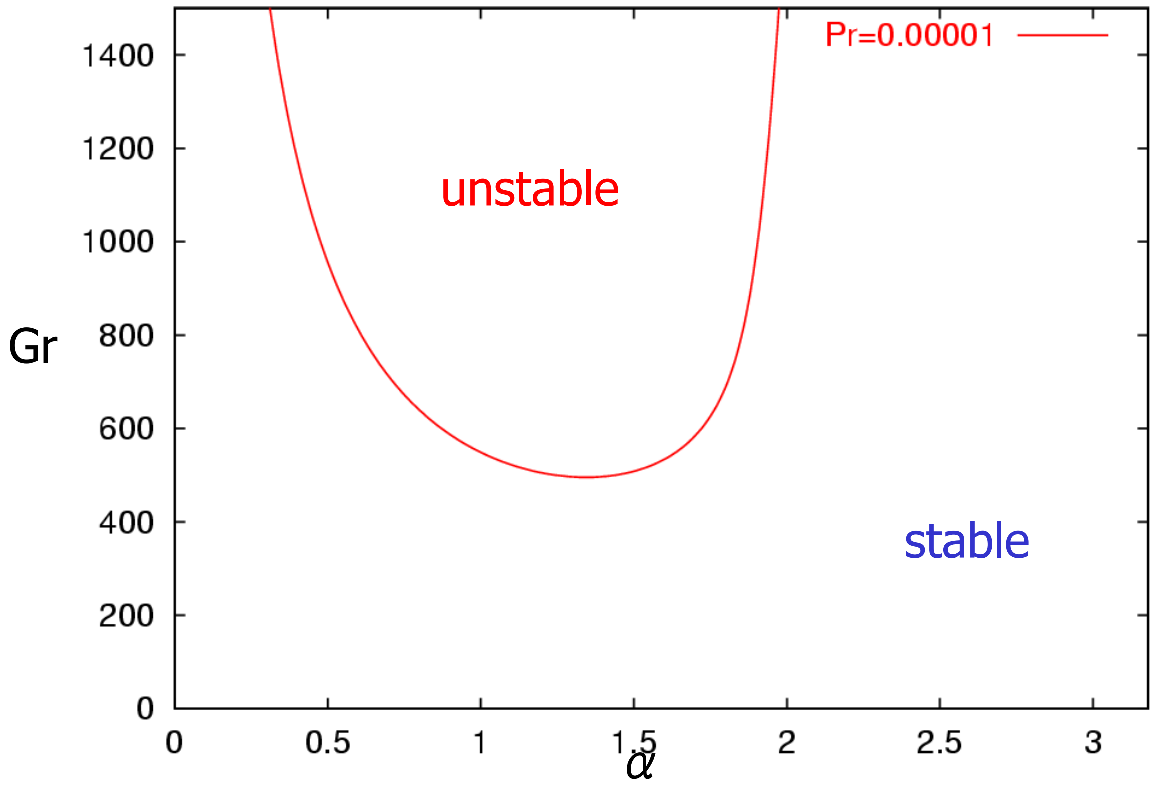

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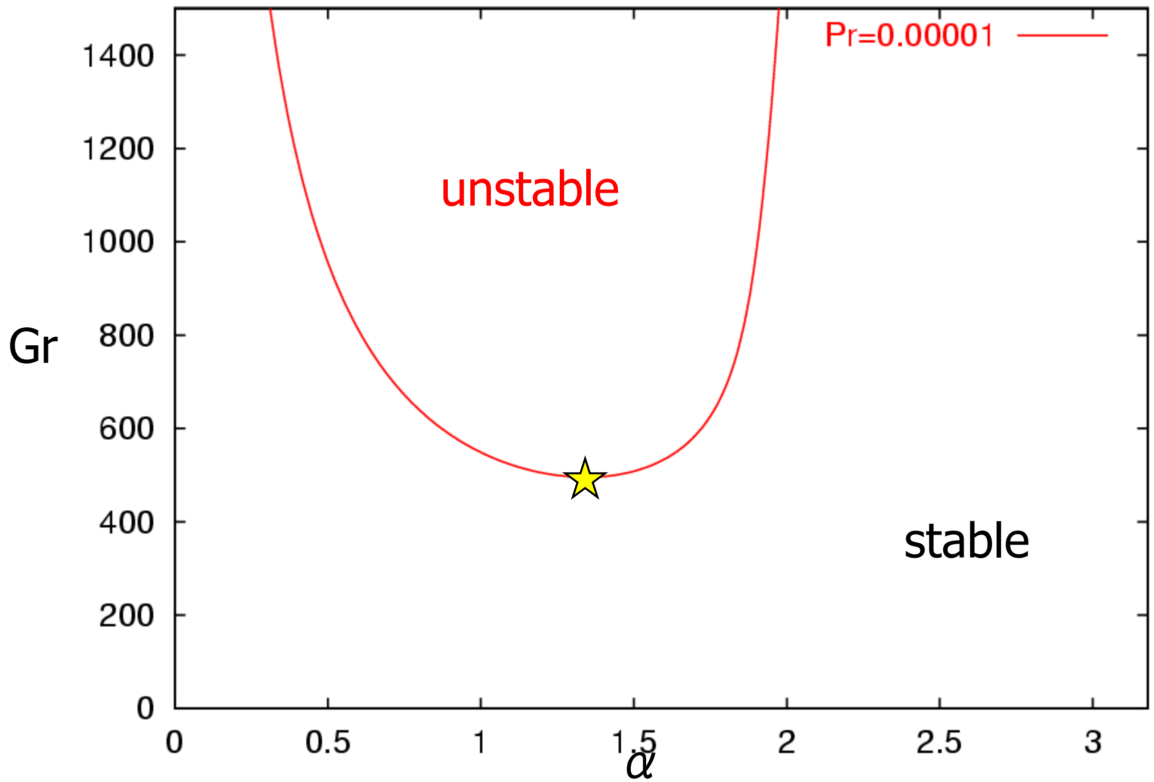

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Linear analysis

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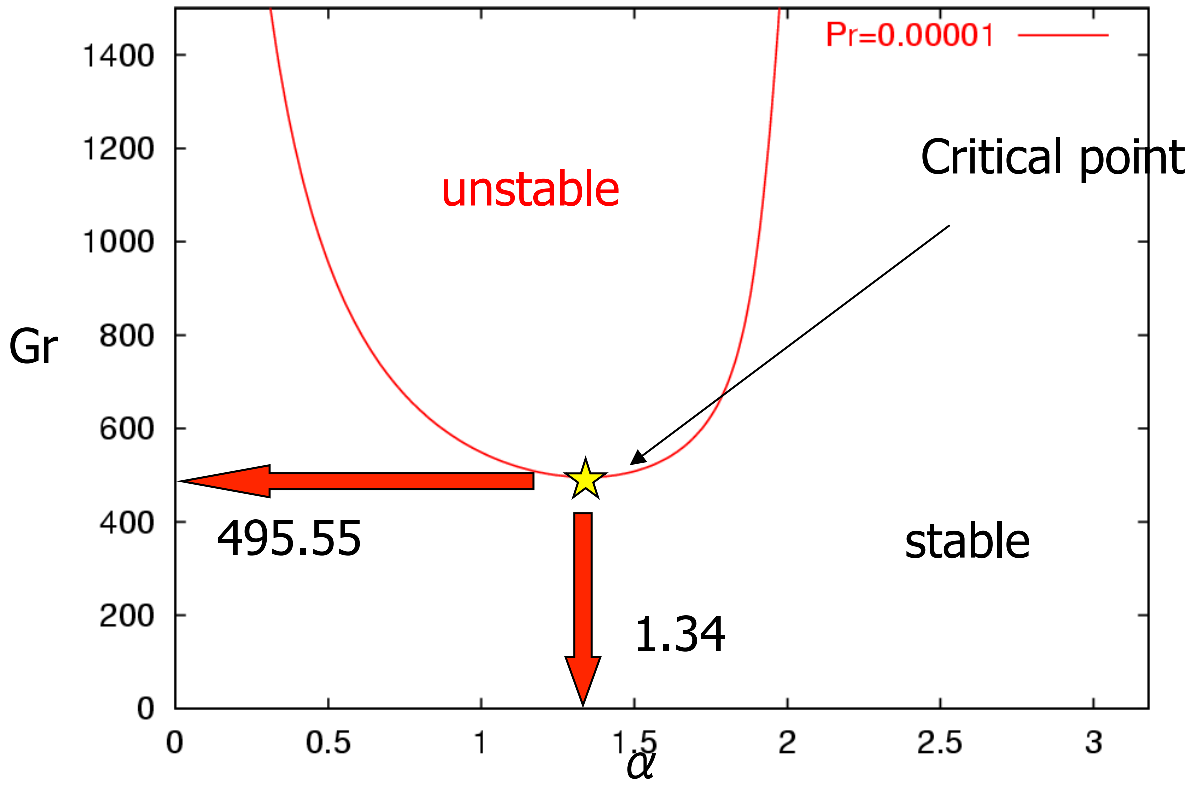

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Summary

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- You will analyse and present your results for a general audience
- Explain in your description the method as thoroughly as possible
- Draw appropriate graphs as for the example that these slides provide.
- By drawing the curves, provide the critical values of wavenumber and Grashof number for your selected case (as in the example given)
- Explain the methodology that you used
- Derive your summary for your study

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Final

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For more resources and information about turbulence, please visit webpage dedicated to ATM2BT at www.eu-atm2bt-project.org

THE END

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